Zentrum für Europäische Integrationsforschung
Center for European Integration Studies
Rheinische Friedrich-Wilhelms-Universität Bonn

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The Excess Volatility of Foreign Exchange Rates: Statistical Puzzle or Theoretical Artifact?

Working Paper

B99-01
1999

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August 1998, *Current Version* January 24, 1999

Abstract

The inability to reconcile observed levels of foreign exchange rate volatility with predictions derived from rational expectations models represents one of the most persistent challenges in international finance. This paper shows that such excess volatility puzzles arise from informational assumptions by contrasting exchange rate equilibria under two different hypotheses: rational expectations and their generalization, rational beliefs. Under the latter agents hold data rather than model consistent expectations requiring learning and inference. Uncertainty arises endogenously as agents with diverse beliefs might trade even in the absence of new information. An analysis of currency volatility mechanisms now reveals that excess volatility is a theoretical consequence of rational expectations' structural knowledge assumptions. Markets only transmit volatility from exogenous variables to exchange rates without any amplification mechanism. Hence, rational expectations equilibria provide a lower volatility bound on more general exchange rate processes solving the excess volatility puzzle in terms of endogenous volatility generation. Finally, the results are applied to explore the structure of currency crises as short-lived rational deviations from economic fundamentals.

*This paper written while I was visiting at ZEI, Bonn University is based on earlier work circulated as "Foreign Exchange Rates under Alternative Expectational Paradigms: A Resolution of the Excess Volatilty Puzzle." Stimulating discussions with Chin-shan Chuang, Bob Flood, Thomas Gehrig, Peter Isard, Lutz Kilian, Mordecai Kurz, Martin Schneider, Sunil Sharma, Jürgen von Hagen, Volker Wieland and Jeff Zwiebel are gratefully acknowledged. I particularly thank Jürgen von Hagen and ZEI, Bonn University, for the kind hospitality during summer 1998. I also benefitted from comments and suggestions of seminar participants at the IMF, the Deutsche Bundesbank, Indiana, Freiburg and Bonn. The earlier paper was presented at the 1997 European Meetings of the Econometric Society, the 1997 EFA meetings, the 1997 EEA meetings. Special thanks go to Lars T. Nielsen, the EFA discussant. All errors, omissions or misinterpretations are solely my responsibility.

1. Introduction

The statistical properties of foreign exchange rates continue to pose a serious theoretical challenge. [13, Flood and Taylor 1996], [15, Frankel and Rose 1994] and [26, Meese 1990] document the general failure of different model classes to generate sufficient amounts of exchange rate volatility in terms of economic fundamentals. Other statistical anomalies such as time varying risk premia or the forward rate bias surveyed in [17, Froot and Thaler 1990] cast further doubt on the validity of standard exchange rate models. While the former might explain the latter risk premia have to be disproportionately variable in comparison to the volatility of fundamentals as analyzed in [2, Bekaert 1995] and [3, Bekaert 1996]. In an attempt to resolve the tension between empirical facts and model predictions this paper examines the theoretical foundations of excessive exchange rate volatility by analyzing learning and diversity of beliefs in foreign currency markets.

The one common feature that most exchange rate models share is their reliance on rational expectations. In particular, they all assume that agents know and agree on the structure of the true model. However, a growing body of empirical evidence suggests that such assumptions might be inappropriate for foreign currency markets. [24, Taylor 1995] makes the general case against homogeneous beliefs in exchange rate determination while [14, Frankel and Froot 1987] provide extensive evidence for heterogeneous expectations in foreign currency markets. Furthermore, the survey by [28, Takagi 1991] on market held exchange rate expectations raises serious doubts about the rationality of such heterogeneous beliefs. In his study on sterling-dollar exchange rates 1981-84 [10, Evans 1986] also identifies disequilibrium expectations whereas [17, Froot and Thaler 1990] report that the bias in interest rate differentials is almost entirely due to a bias in exchange rate expectations.

While rational expectations insure model consistent beliefs and analytic tractability they also impose heavy statistical restrictions on equilibrium processes. The latter is a result of the severe informational assumption that agents have full knowledge of the economy and, more importantly, its underlying stochastic structure. Structural knowledge helps to solve for the endogenous variables in terms of exogenous ones so that the only uncertainty in equilibrium comes from the systematic variation of economic fundamentals. Hence, one can not increase the variability of endogenous variables (exchange rates) without first increasing the variability of exogenous ones. This observation explains why efforts to modify common rational expectations models of foreign currency markets such as [25, Manuelli and Peck 1990] and [3, Bekaert 1996] have been only partially successful. It also suggests a new line of attack on foreign currency puzzles.

Instead of solving different models for equilibrium exchange rate processes under the same beliefs assumption this paper compares the statistical properties of equilibria in a simple version of the same two-country [23, Lucas 1982] model under alternative expectational paradigms. Rational expectations represent homogeneous beliefs while diversity in opinion and learning are modeled in terms of rational beliefs as developed in [19, Kurz 1991]. This approach lends itself to an investigation of the implied volatility mechanisms because rational beliefs encompass rational expectations as a special case. The former simply generalizes the latter's rationality criterion of model consistent expectations to data consistent beliefs with learning . Consequently, rational beliefs lead to equilibria in which agents hold divergent, yet rational views. Equilibrium realizations then have differential information content so that contrary to rational expectations agents may trade even in the absence of new information. The interaction of learning, diversity in beliefs and trading now generates endogenously uncertainty and resolves the excess volatility puzzles of exchange rates.

Decomposing the rational beliefs variance into the rational expectations variance and an endogenous uncertainty term identifies the source of excess volatility: aggregated learning behavior. Learning behavior or its absence, in turn, characterize volatility mechanisms. Rational expectations correspond to the deterministic transmission of exogenous uncertainty while rational beliefs endogenously amplify and generate uncertainty. Regarding first moments, exchange rate processes converge to the same asymptotic mean under both expectational hypotheses. However, endogenous uncertainty permits temporary deviations from fundamentals with correspondingly higher rational beliefs variance. The conclusion is immediate: the mystery of the missing exchange rate volatility is a theoretical artifact of rational expectations and unreasonable informational assumptions.

This paper contributes to the growing literature that attempts to explain foreign exchange market anomalies in terms of expectational errors. Its main innovation lies in avoiding *ad hoc* beliefs specifications by casting rationality in terms of data consistent learning[1]. It turns out that this weakening of rationality is just sufficient to identify the volatility transmission mechanism as the deeper source of excess volatility puzzles. The next section presents an informal introduction to rational beliefs as a generalization of rational expectations. Section 3 develops a simple model of foreign currency markets, derives rational expectations and beliefs equilibria and compares their statistical properties. Section 4 analyzes exchange rate volatility in the light of the derived statistical properties of rational beliefs equilibria and introduces the concept of endogenous uncertainty. All the proofs and a summary of rational beliefs are relegated to the Appendix.

2. From Rational Expectations to Rational Beliefs

Modeling decision making under uncertainty requires the judicious specification of beliefs about the stochastic environment. In economics, the paradigm of rational expectations (RE) has been so successful because it avoids logical contradictions and model inconsistencies by imposing an individual rationality constraint on admissible beliefs. The essence of rational expectations equilibria consists in insuring that realizations verify beliefs by making outcomes consistent with the underlying economic model. If equilibrium realizations systematically contradicted agents' expectations acting upon such equilibria would violate all notions of rationality. Not only do

[1]This approach is similar in spirit to [22, Lewis 1989] where agents learn about regime changes - one-time, unobservable shocks - whose occurrence are only gradually accepted. The shock effects persist and excess returns are not traded away as one would expect in a learning environment.

agents never learn from their mistakes; worse, knowing the economy's structure they nevertheless persist in their suboptimal behavior in full knowledge of their own irrationality.

Instead, expectations are said to be rational if calculated with respect to the true conditional probabilities as implied by the underlying economic structure. A rational expectations equilibrium (REE) consists of a price process and a collection of conditional probabilities such that markets clear under these prices when agents solve their optimization problems for the given conditional probabilities. Corresponding to fixed points of mappings from prices to probability distributions their computation is greatly facilitated by assuming that agents know the true model of the economy. Under this assumption of structural knowledge agents bring about equilibria whose realizations will not contradict their expectations. Put differently, equilibrium realizations are what agents think they should be: agents' ex ante expectations are verified ex post by the data.

While insuring rational behavior and analytic tractability the structural knowledge hypothesis poses several problems. First, it assumes unlimited information acquisition and processing capacities. Secondly, it severely limits admissible learning processes that conform to notions of rationality. Most importantly, structural knowledge makes it very difficult to model diversity of opinion while preserving rationality. This point is often obscured by the very success of rational expectations in modeling heterogeneous information although in many models one still has to resort to agent categories (noise or liquidity traders) whose actions can not be fully accounted for by the model. As a direct consequence of the severe informational requirements rational expectations impose strong statistical restrictions on equilibrium process as will be shown in the context of foreign currency markets.

The theory of rational beliefs[2] (RB) proceeds by asking what probabilistic assessments agents could reasonably hold in light of their observations. In a dynamic setting agents observe a vector of economic fundamentals x_t. Not knowing the true structure of the economic model they compute their probabilistic assessment of the environment as frequencies from an arbitrarily long but finite history of data. In order to assess a finite dimensional event S and a history $x := (x_0, \dots, x_{n-1})$ they construct empirical distribution functions

$$m_n(S)(x) := n^{-1} \sum_{t=0}^{n-1} 1_S(x_t)$$

where 1_S is the indicator function of the event[3] counting how many times it has occurred in the sample. Trying to discover the true probability of an event agents calculate their assessments as the relative frequency of the dynamical system visiting S given that it started at x.

By casting individual rationality in terms of an inference problem RB replace the RE criterion of model consistency with the weaker criterion of data consistency implied by the former. As

[2] Other expectational hypotheses with different rationality criteria are [6, Bossaert 1996] where agents' consistent beliefs are restricted by Bayes' Rule or [1, Arifovic 1996] interpretating bounded rationality in terms genetic algorithms. See Appendix A for a formal summary of rational beliefs.

[3] The indicator function $1_S(x)$ of a set S is defined as $1_S(x) := \begin{cases} 1 & \text{for } x \in S \\ 0 & \text{else} \end{cases}$.

a first plausibility requirement frequencies computed from the observed data[4] have to converge as more and more data become available. Hence, the existence of $n \to \infty \lim m_n (S)(x)$ called stability is a prerequisite for learning, meaningful inference and rationality. In the absence of stability, one arrives at the counter-intuitive situation where more data complicate rather than facilitate inference. By appealing to standard measure theoretic results one can generate probability measures m from these empirical frequencies. This empirical measure summarizes all that can be learned from the past and is, trivially, common to all agents. Notice that knowledge does not equal causal knowledge as the mechanisms generating empirical probabilities are unknown.

Stability embodies a notion of restricted learnability since agents can not perfectly learn their environment which is the foundation of rational expectations and structural knowledge hypotheses. To qualify as rational agents' subjective beliefs are restricted by two further conditions: data compatibility and non-degeneracy of subjective probability assessments. The former requires that frequencies computed under a rational belief Q generate the empirical measure. Data compatibility insures that beliefs and realizations are consistent with each other. Testing their assessments on the basis of observations they would not reject a rational belief. The latter is a plausibility criterion: if an event is observed to have occurred and, therefore, has positive empirical probability under m its occurrence should not be excluded under agents' beliefs requiring a positive probability, too.

A fundamental result in [19, Kurz 1991] links empirical observation to the rationality of beliefs. If a probability measure Q is a rational belief, then it can be represented as the convex combination of two probabilities:

$$Q = \lambda m + (1 - \lambda) P$$

where m is the empirical measure, P is a subjective probability and $0 < \lambda \leq 1$. As a direct consequence of stability the economy appears stationary to the agents under the empirical measure. Hence, any rational belief can be expressed as a weighted average of the common stationary assessment and a subjective one of the non-stationary aspects. While m represents information available to all agents P captures both the evolution of the economy and rare, infrequent events in the subjective assessment. Put differently, beliefs are rational if they can be expressed as a convex combination of common knowledge and the subjective likelihood of rare events. Moreover, it can be shown that both m and P are mutually exclusive (singular): rational beliefs decompose each event into a frequent, stationary and rare, non-stationary part.

The stage is now set to define a rational beliefs equilibrium (RBE) in analogy with a rational expectations equilibrium: it is a stable economic system with a true, possibly unknown probability, a set of probabilities which are rational beliefs with respect to the true probability and a price process so that agents attain optimal allocations under their beliefs and market clearing. In equilibrium, realizations conform to subjective conditional probabilities whereas in REE they are

[4]Example: over a period of ten years, the (conditional) probability of observing a rise in the Dow-Jones Industrials index after a fall can be approximated as a frequency by $\frac{\#\{\text{rise following fall in DJI over 10 years}\}}{2{,}500}$.

consistent with true conditional probabilities. Contrary to REE prices can not be recovered from beliefs: given the true probabilistic structure equilibria typically can result from many beliefs systems as long as beliefs and equilibrium realizations are compatible. The following proposition summarizes the rationality conditions used to calculate rational belief equilibria:

Proposition 2.1 (Rational Belief Restrictions). *Holding a rational belief Q_k implies*

1. *decomposition: for $\lambda_k \in (0,1]$, the empirical measure m and the subjective one P_k*
$$Q_k = \lambda_k m + (1-\lambda_k) P_k; \tag{2.1}$$
2. *orthogonality: m and P_k are mutually singular;*
3. *stability: the economic system is stable under both Q_k and P_k and $m_{Q_k} = m = m_{P_k}$ so that all asymptotic moments exist under the empirical measure m.*

Consider a stable dynamic economy where agents do not possess structural knowledge. Information corresponds to past realizations of the observable economic variables as contained in information sets $\{\mathcal{G}_t\}_t$. According to Equation (2.1) some agent k holding rational belief Q_k computes her forecast of Y_{t+1} at t conditional on information $\mathcal{G}_t$ as

$$E_{Q_k}[Y_{t+1}|\,\mathcal{G}_t] = \lambda_k E_m[Y_{t+1}|\,\mathcal{G}_t] + (1-\lambda_k) E_{P_k}[Y_{t+1}|\,\mathcal{G}_t]$$

since $Q_k = \lambda_k m + (1-\lambda_k) P_k$. In RE agents know the true probability Π so that they compute assessments under $\Pi = Q_k$ as $E_\Pi[Y_{t+1}|\,\mathcal{G}_t]$. This is the encompassing property of RB required in the sequel: rational beliefs contain rational expectations under the special case of structural knowledge. Furthermore, one can see how RB offer a natural way to model heterogeneity of beliefs: while in the long run beliefs have to converge to the common forecast $E_m[Y_{t+1}|\,\mathcal{G}_t]$, the second term leaves quite some room for temporary rational disagreement.

Heterogeneous beliefs generate the statistically interesting properties of RBE because stability only dictates that the non-stationary part of expectations even out over time. In the short run almost any pattern is possible provided that it is not too repetitive; otherwise, it might contribute mass to long-term averages violating rationality. An intriguing statistical dichotomy arises. Over short horizons, Q_k, i.e., P_k and λ_k, matter so that equilibrium realizations might exhibit a high degree of volatility and non-stationarity. But the same data's asymptotic behavior under the empirical measure m might appear stationary because standard asymptotic theory applies. In the absence of structural knowledge estimation and inference have to rely on empirical distributions. Since beliefs have to be rational *ex ante* agents can rationally disagree about the non-stationarity of the world. Statistical theory will not help them to discriminate between competing models since both might be compatible with the data and, therefore, can not be rejected.

A further defining characteristic of RBE is endogenous uncertainty. Agents with heterogeneous beliefs might trade in response to equilibrium realizations even without new information simply because they draw different inferences from the data. For a given level of exogenous

t_0	t_1	t_2
State Realization	*Securities Trading*	*Goods Acquisition*
$\tilde{x}_t$	Trading in FX, shares and government debt	Purchase of C_t^i

Figure 3.1: Trading Structure

variability, the volatility of endogenous variables rises under RB. By the same token, price processes can temporarily deviate from their true fundamentals. In the long run such deviations have to disappear by stability and are rare in the probabilistic sense. But at any given point phenomena associated with financial crises such as (speculative) bubbles, panics and currency attacks might occur with positive probability. If agents' beliefs exhibit sufficient correlation they become self-sustaining and ever more likely. Local crises are then defined as episodes of price changes which do not coincide with a corresponding change in fundamentals. Global crises, on the other hand, result from the same assumptions as in REE.

3. Exchange Rate Determination

Two countries $i = 1, 2$, each with its own currency, specialize in the production of only one consumption good X_t^i. Production takes place in each period t according to some exogenous stochastic process. Governments issue new currency $\left(M_{t+1}^i - M_t^i\right)$ at time t in order to acquire domestic consumption goods, which have no incidence on agents' well being ("wastes resources"). Hence, the economic fundamentals are the observables in the economy

$$\tilde{x}_t := \left(X_t^1, X_t^2, M_{t+1}^1, M_{t+1}^2\right).$$

Initial wealth is evenly distributed between the residents of both countries. At the outset of each period t, which is further divided into three subperiods, economic fundamentals are realized. Households in country i then trade in foreign currency with their governments (central banks). In the next subperiod households purchase consumption goods where goods produced in country i can only be purchased with i's currency. Hence, the trading structure summarized in Figure 3.1 implements a cash-in-advance constraint. The government purchases all goods domestically and holds foreign currency only in order to trade with households in the country. In the last subperiod, households collect dividends from the firms they own shares in.

In both countries there is an uncountable number of identical consumers on the unit interval with preferences[5]

$$U^{k,i} = E_0 \left\{ \sum_{t=0}^{\infty} \delta^t \left[\beta^i \log \left(C_t^i - \bar{c}^i \right) + \beta^j \log C_t^j \right] \right\}, i \neq j$$

where $C_t^{k,i}$ is time t consumption of good X_t^i by agent $k \in [0,1]$. For incomes m_t^k and prices P_t^i individual demands for the foreign good are $C_t^{k,j} = \beta^j \left[\frac{m_t^k}{P_t^j} - \frac{P_t^i}{P_t^j} \bar{c}^i \right]$. Under the standard assumption of purchasing power parity foreign currency demand is linear in the exchange rate $e_t := \frac{P_t^i}{P_t^j}$, $i \neq j$:

$$d_t^k := C_t^{k,j} = \alpha_t^k - \beta e_t$$

with $\alpha_t^k := \beta^j \frac{m_t^k}{P_t^j}$ and $\beta := \beta^j \bar{c}^i$. The cash-in-advance constraint requires agents to choose their foreign currency demand before the exchange rate e_t, the price at t of one unit of foreign in domestic currency, is known. Given agent k's forecast under probability assessment Q_k expected utility maximization results in demands

$$d_t^k = \alpha_t^k - \beta E_{Q_k} \left[e_t | \mathcal{G}_t \right], \quad \alpha_t^k, \beta > 0$$

where the $\mathcal{G}_t$ are the information sets generated by the fundamentals $\tilde{x}_t$ and e_{t-1}. Aggregate demand becomes

$$d_t := \int_0^1 d_t^k dk = \alpha_t - \beta \int_0^1 E_{Q_k} \left[e_t | \mathcal{G}_t \right] dk + \eta_t$$

where the above integrals are assumed to exist, $\alpha_t := \int_0^1 \alpha_t^k dk$ and η_t is a sequence of mean 0 random variables representing global demand shocks. Suppose that aggregated individual demand shocks $\{\alpha_t\}_t$ are a deterministic sequence asymptotically uncorrelated with any past data[6].

In each period, the central bank acting as a market maker supplies foreign currency from its reserves in function of the fundamentals $\tilde{x}_t$. Let the authorities (government) pursue an exchange rate target $\bar{e}_t$ so that monetary policy M_{t+1}^i attempts to minimize $E\left[\left| \frac{\bar{e}_t}{e_t} - 1 \right| \middle| \mathcal{G}_{t-1} \right]$. Actual realizations are bounded by some function of exchange rates so that $\left| \frac{\bar{e}_t}{e_t} - 1 \right| < \chi(e_t)$. Under the quantity theory equations $P_t^i = \frac{M_{t+1}^i}{X_t^i}$ derived from goods and money market equilibrium exchange rates are $e_t = \frac{P_t^i}{P_t^j} = \frac{M_{t+1}^i}{M_{t+1}^j} \frac{X_t^j}{X_t^i}$. Approximating $\log \chi(e_t)$ by some linear combination of fundamentals $x_t^{\mathrm{T}} \tilde{h}$ where $x_t = \log \tilde{x}_t$ and observing that $\log \left| \frac{\bar{e}_t}{e_t} - 1 \right| \cong \bar{e}_t - e_t$ implies for the unobserved mean 0 error ϵ_t

$$\bar{e}_t = e_t + x_t^{\mathrm{T}} \tilde{h} + \epsilon_t.$$

[5]In this specification the import C^j is less of a necessity than C^i for residents of country i, $i \neq j$ since a minimum of $\bar{c}^i$ of the domestic good needs to be consumed, i.e., $C^i \geq \bar{c}^i$.

[6]Both assumptions are for ease of exposition only; demand shocks could be, e.g., Markovian.

Managing exchange rates and currency reserves generates revenues and costs. The central bank stands ready to buy or sell foreign currency according to the quasi-linear objective function[7] $g_t(x_t) := \bar{e}_t q_t - \frac{1}{2\gamma} q_t^2$, $\gamma > 0$. The quadratic term captures the costs associated with achieving the target rate such as, e.g., the danger of exhausting reserves. Substituting in for $\bar{e}_t$ now results in

$$g_t(x_t) = \left(e_t + x_t^{\mathrm{T}} \tilde{h} + \epsilon_t\right) q_t - \frac{1}{2\gamma} q_t^2$$

so that foreign currency is optimally supplied according to

$$s_t = \gamma e_t + x_t^{\mathrm{T}} \tilde{h} + \gamma \epsilon_t$$

with $h = \gamma \tilde{h}$. Market clearing in each period t requires $d_t = s_t$ so that e_t satisfies

$$e_t = \left(a_t + x_t^{\mathrm{T}} h\right) + b \int_0^1 E_{Q_k}\left[e_t | \mathcal{G}_t\right] dk + \xi_t \tag{3.1}$$

with $a_t = \frac{\alpha_t}{\gamma} > 0$, $b = -\frac{\beta}{\gamma} < 0$, $\xi_t = \frac{(\eta_t - \gamma \epsilon_t)}{\gamma}$. Exchange rate uncertainty comprises two terms: systematic shocks $\left(a_t + x_t^{\mathrm{T}} h\right)$ and ξ_t, the idiosyncratic one.

So far, no expectational hypothesis has been specified. Under structural knowledge, agents compute their expectations with respect to the true probability $\Pi = Q_k$, $\forall k$. Taking conditional expectations[8] in Equation (3.1) and solving for the equilibrium price process motivates:

Proposition 3.1 (REE). *The exchange rate process e_t is a* rational expectations equilibrium *if it satisfies model consistent beliefs $\Pi = Q_k$, $\forall k \in [0,1]$ market clearing and optimality at all t so that*

$$e_t^{REE} = \left(a_t + x_t^T h\right) + \frac{b}{(1-b)} \left(a_t + x_t^T h\right) + \xi_t = (1-b)^{-1} \left(a_t + x_t^T h\right) + \xi_t. \tag{3.2}$$

In light of Proposition 2.1 the stability requirement of rational beliefs implies that all asymptotic moments exits: the economy is stable. Such convergence does not rely on independence and some LLN but is a pure consequence of data consistency and learnability. Indeed, the present framework can accommodate almost any exogenous stochastic specification which is also borne out by the rational beliefs equilibrium:

Proposition 3.2 (RBE). *The exchange rate process e_t is a* rational beliefs equilibrium *if it satisfies for the collection of beliefs $\{Q_k\}_{k \in [0,1]}$ data consistency with common empirical probability m, market clearing and optimality at all t under Π; for $\lambda := \int_0^1 \lambda_k dk$ and $e_t^P(\mathcal{G}_t) := \int_0^1 (1-\lambda_k) E_{P_k}[e_t | \mathcal{G}_t] dk$ the rational beliefs equilibrium exchange rate process is*

$$e_t^{RBE} = \left(a_t + x_t^T h\right) + b\lambda (1-b)^{-1} \left(a + x_t^T h\right) + b e_t^P(\mathcal{G}_t) + \xi_t. \tag{3.3}$$

[7] Or some monotone transformation thereof: one could have taken the central banks to be risk averse with $G_t^i = -\exp\left[-\rho^i \left[\bar{e}_t q_t - c(q_t)\right]\right]$, $\rho^i > 0$ with cost function c.

[8] By the cash-in-advance constraint the demand for foreign currency is chosen before the exchange rate e_t becomes known but after the state variables have been realized: x_t is contemporaneous information.

Proof. See Appendix. ■

The equilibrium exchange rate processes (3.2) and (3.3) are of the same form: fundamentals + aggregated forecasts + exogenous shock. The differences arise in the forecasting term: structural knowledge $Q_k = \Pi$ or heterogeneous beliefs. As a consequence of the former, exchange rates are a deterministic function of fundamentals under RE. But in a RBE the true probability Π and the collection of rational beliefs Q_k do not need to coincide: they are only required to be compatible. Consequently, heterogeneous beliefs enter the exchange rate process through subjective non-stationarities $e_t^P(\mathcal{G}_t)$. RBE exchange rates are random functions of fundamentals and each set of beliefs generates a different process. Only the stationary, empirical part is common while the interaction of the subjective, non-stationary parts creates interesting statistical properties.

Consider the following special RBE as a benchmark where all agents believe that the environment is stationary, i.e., $Q_k = m,\ \forall k$, so that $\lambda_k = 1 = \lambda$ and Π is compatible with this belief (at most, only slightly non-stationary). The equilibrium exchange rate process under m induced by Π becomes

$$e_t^{RBE^*} = \left(a_t + x_t^{\mathrm{T}} h\right) + b\,(1-b)^{-1}\left(a + x_t^{\mathrm{T}} h\right) + \xi_t. \tag{3.4}$$

Comparison of (3.2) and (3.4) reveals how similar the two expectational paradigms are: the exchange rate process only differs in the a term of the forecast. Rationality in terms of model consistency requires agents to forecast demand shocks exactly (a_t) while data consistent rationality leads to averaging in forecasts by stability (a). Given that rational expectations are a special case of rational beliefs the close resemblance of exchange rate processes for homogeneous, stationary beliefs should not come as a surprise.

Stability yields the first fundamental insight into the statistical properties of the respective equilibrium exchange rate processes: first moments are asymptotically equivalent due to the transitory nature of subjective non-stationarities.

Proposition 3.3 (Asymptotic Means). *Long-term sample means of exchange rates converge to a common average under the two expectational paradigms; for* $e := (1-b)^{-1}\left(a + x^T h\right)$

$$T \to \infty \lim T^{-1} \sum_{t=0}^{T-1} e_t^{REE} = e = T \to \infty \lim T^{-1} \sum_{t=0}^{T-1} e_t^{RBE}.$$

Proof. See Appendix. ■

The convergence of exchange rates to true means is a general result that does not depend on any model specific features. A consequence of learning, it hints at the problems carrying out inference and paradigm testing on foreign exchange data. Attempts to discover the true distribution Π from long-term averages are futile unless agents happen to hold the same belief $Q_k = \Pi$: they only calculate approximations to the stationary measure m. But beliefs $Q_k = \Pi$ correspond to RE with structural knowledge where agents do not need to infer Π to start with. In its absence, statistical theory relying on knowledge of the true model can not be used to distinguish between competing views of exchange rate determination as represented by heterogeneous RB. Data consistency is a weaker rationality criterion precisely because competing models - beliefs - are less easily rejected.

4. Foreign Currency Volatility Mechanisms

While the asymptotic convergence of sample means is reassuring, even expected, it raises the question how expectations, diversity of beliefs and forecasting affect statistical properties. If first moments do not statistically distinguish equilibrium exchange rate processes perhaps second moments can. From Proposition 2.1 agents differ in their assessment of non-stationarities P_k and their weight λ_k. In the RBE exchange rate process (3.3) the interaction of the true process and the aggregated subjective parts of beliefs feeds through into data non-stationarity. Realizations verify the heterogeneous priors about model structure and non-stationarities ex-post. Hence, aggregating heterogeneous beliefs under data consistent rationality yields the additional degree of statistical freedom rational expectations model lack.

Consider the special case of homogeneous, stationary beliefs: Equation (3.4) conserves the a_t term under the beliefs $Q_k = m$, $\forall k$. Comparison with the REE (3.2) highlights the impact of beliefs on exchange rates: the difference

$$e_t^{RBE^*} - e_t^{REE} = b\,(1-b)^{-1}\,(a - a_t)$$

reflects true non-stationarity in the data induced by forecasting without structural knowledge. Over time, such rational forecasting mistakes vanish in the mean by Proposition 3.3. However, in the meantime they may generate considerable variability in exchange rates. Conversely, in the special case of a truly stationary environment, i.e., $a_t = a$, where agents hold non-stationary RB, data consistent forecasting mistakes follow as

$$e_t^{RBE^{**}} - e_t^{REE} = b\,(\lambda - 1)\,(1-b)^{-1}\left(a + x_t^{\mathrm{T}} h\right) + b \int_0^1 (1-\lambda_k)\, e_t^{P_k}(\mathcal{G}_t)\,. \tag{4.1}$$

Here, the aggregate effects of beliefs give rise to a non-stationary equilibrium price process in a stationary environment through the last term. Finally, if the environment were stationary and agents held stationary beliefs, i.e., $a_t = a$ and $\lambda_k = 1 = \lambda$, then Equation (4.1) shows that the exchange rate process is stationary and the two equilibria coincide.

In REE models, markets transform realizations of economic fundamentals into exchange rate volatility as agents react to perfectly anticipated changes in the environment.

Proposition 4.1 (REE Variance). *Long-term sample variances of exchange rates under rational expectations converge to*

$$\sigma_{REE}^2 = (1-b)^{-2}\left(\sigma_a^2 + \sigma_x^2\right) + \sigma_\xi^2. \tag{4.2}$$

Proof. See Appendix. ■

The volatility of economic fundamentals x_t, non-stationary trend variables a_t and random shocks ξ_t directly determine exchange rate volatility. In a stationary world $a_t = a$ and $\sigma_a^2 = 0$ so that the only way to increase the variance of REE exchange rates consists in assuming higher variances of economic fundamentals σ_x^2 or shocks. Similarly, in the non-stationary case with $\sigma_a^2 < \infty$ exchange rate volatility only varies with fundamentals including shocks.

In RBE, markets endogenously generate volatility. Feedback effects from equilibrium realizations and fundamentals back to exchange rates via heterogeneous beliefs characterize the stochastic properties of equilibria. Agents interpret realizations in light of their own beliefs so that exchange rates signal different information about the environment to different market participants. As a result, trading can occur even in the absence of new information ($a_t = a$, say). By acting upon equilibrium realizations under their subjective beliefs agents amplify both non-stationarity and volatility. Beliefs and actions rather than economic fundamentals propagate uncertainty in foreign currency markets causing complementary volatility:

Proposition 4.2 (RBE Variance). *Long-term sample variances of exchange rates under RBE converge for aggregate forecasting functions* $H_t := \int_0^1 E_{Q_k}\left[e_t \middle| \mathcal{G}_t\right] dk$ *to*

$$\begin{aligned} \sigma_{RBE}^2 &= (1-b)^{-2}\left[\sigma_a^2 + \sigma_x^2 + b^2\left(\sigma_H^2 + \sigma_a^2 + \sigma_x^2\right)\right] + \sigma_\xi^2 \\ &= (1-b)^{-2}\left(\sigma_a^2 + \sigma_x^2\right) + \sigma_\xi^2 + b^2(1-b)^{-2}\left(\sigma_H^2 + \sigma_a^2 + \sigma_x^2\right); \end{aligned} \tag{4.3}$$

in the special $Q_k = m$ *case of (3.4),* $\sigma_{RBE^*}^2 = \sigma_a^2 + (1-b)^{-2}\sigma_x^2 + \sigma_\xi^2$.

The above expression admits a decomposition of exchange rate volatility into its exogenous and endogenous components. Exogenous uncertainty represented by the variance of fundamentals including exogenous shocks contributes $(1-b)^{-2}\left(\sigma_a^2 + \sigma_x^2\right) + \sigma_\xi^2$ to exchange rate variability. Comparison with (4.2) identifies exogenous variability as the REE variance of exchange rates. The need to form a subjective assessment of the underlying processes induces added variance in the form of persistent aggregate forecasting mistakes with variance $b^2(1-b)^{-2}\left(\sigma_H^2 + \sigma_a^2 + \sigma_x^2\right)$. Composed of forecasting uncertainty σ_H^2 and trading induced variability of fundamentals $\sigma_a^2 + \sigma_x^2$, the RBE complement captures endogenous uncertainty stemming from lack of structural knowledge. Even with homogeneous, stationary beliefs structural uncertainty about demand shocks feeds through to increase exchange rate volatility in the special case (3.4).

The preceding results illustrate the long-run behavior of exchange rate volatility under the two expectational paradigms. Comparing the volatility structure of exchange rate processes in Equations (4.2) and (4.3) confirms that the asymptotic REE variance provides a lower bound on exchange rate variability under rational beliefs. In the short-run, RBE places much less restrictions on volatilities than REE: the non-stationary part of beliefs can induce any value of the conditional variance $\sigma^2(\mathcal{G}_t)$ over a finite time period. Taken together these observations resolve the excess volatility puzzle in terms of heterogeneous beliefs and data consistent learning:

Proposition 4.3 (Variance Bounds). *The REE variance provides a lower bound for long and short-term exchange rate variability across belief systems:*

$$\sigma^2_{REE} \leq \sigma^2 (\mathcal{G}_t) \leq \sigma^2_{RBE}.$$

Volatility - variance - is a reflection of both the degree of knowledge about the source of uncertainty and the uncertainty itself. Hence, it is intuitive that REE equilibrium exchange rate processes should provide a lower bound on exchange rate variability. By the structural knowledge hypothesis there is no uncertainty as to the source of volatility or its stochastic properties. Only the randomness of the economic fundamentals feeds into the equilibrium variance processes as illustrated by Equation (4.2). In this sense, the volatility puzzle of exchange rates under rational expectations is hardly a puzzle at all: the low variance is a defining statistical characteristic of REE exchange rate processes. Any endogenous volatility generation or amplification mechanism is missing: exchange rate volatility is a deterministic function of exogenous volatility.

But then the mystery of the missing volatility is not a statistical puzzle but a theoretical artifact of structural knowledge. It originates more from the choice of expectational paradigm than any other modeling choices. Since REEs are calculated in terms of exogenous variables, only fundamental uncertainty, not uncertainty about the source of the randomness drives exchange rate volatility. In the long run, exchange rate means converge to a common asymptotic value across paradigms. In the short run, temporary deviations from economic fundamentals, impossible under rational expectations, may rationally occur in rational beliefs equilibria. While such effects can not persist by data consistent rationality they leads to higher theoretical predictions of exchange rate volatility that better agrees with empirical facts.

As an illustration of exchange rate statistics consider the following ARX(n) beliefs system

$$E_{Q_k} [e_t | \mathcal{G}_t] = (1-b)^{-1} \left[\sum_{l=0}^{n} \theta^k_{t-l} e_{t-l-1} + x_t^{\mathrm{T}} h \right]$$

where the true probability Π and weights $\left\{\theta^k_{t-l}\right\}_k$ are chosen to be data consistent and the exchange rate exhibits mean-reversion. The asymptotic variance follows from Proposition 4.2 as

Corollary 4.4 (ARX(n) Variance). *Under an ARX(n) beliefs system the asymptotic variance is given for aggregate forecasting functions* $\theta_t := \sum_{l=0}^{n} \left(\int_0^1 \theta^k_{t-l} dk \right) e_{t-l-1}$ *by*

$$\sigma^2_{ARX} = (1-b)^{-2} \left(\sigma^2_a + \sigma^2_x\right) + b^2 (1-b)^{-2} \left(\sigma^2_\theta + \left(1 - \frac{2}{b}\right) \sigma^2_a\right) + \sigma^2_\xi.$$

Once again, the $b^2 (1-b)^{-2} \left(\sigma^2_\theta + \left(1 - \frac{2}{b}\right) \sigma^2_a\right)$ term captures the volatility induced by endogenous uncertainty. It also shows how the non-deterministic nature of RBE exchange rates translates into volatility generation rather than REE's transmission from exogenous variables.

Now, consider the general non-stationary forecasting problem individual agents face: knowing the common forecast $E_m[e_t|\mathcal{G}_t]$ they still need to form assessments of a_t and H_t as reflected in

$$e_t^{RBE} = E_m[e_t|\mathcal{G}_t] + (1-b)^{-1}(a_t - a) + b\left\{H_t - (1-b)^{-1}\left(a_t + x_t^{\mathrm{T}}h\right)\right\} + \xi_t.$$

If agents knew the true underlying probability distribution Π the innovation $\nu_t^{\Pi} \equiv \xi_t$ defined by $e_t = E_\Pi[e_t|\mathcal{G}_t] + \nu_t^{\Pi}$ would relate realizations to forecasts. Hence, forecasts under Π become

$$E_\Pi[e_t|\mathcal{G}_t] = (1-b)^{-1}\left\{a_t + x_t{}^{\mathrm{T}}h\right\} + b\left\{H_t - (1-b)^{-1}\left(a_t + x_t^{\mathrm{T}}h\right)\right\}.$$

If $H_t = (1-b)^{-1}\left(a_t + x_t^{\mathrm{T}}h\right)$ - the structural knowledge case - one is back in the REE setting with accordingly low short and long-term variance. In reality, agents do not know Π but forecast under their rational beliefs Q_k. The result are subjective forecasting error ν_t^k in $e_t = E_{Q_k}[e_t|\mathcal{G}_t] + \nu_t^k$. Defining the subjective forecasting deviations by $\zeta_t^k := E_{Q_k}[e_t|\mathcal{G}_t] - E_\Pi[e_t|\mathcal{G}_t]$ it follows from the preceding that

$$\zeta_t = \int_0^1 \zeta_t^k dk = (1-b)\left\{H_t - (1-b)^{-1}\left(a_t + x_t^{\mathrm{T}}h\right)\right\}.$$

Aggregate forecasting deviations ζ_t summarize several implications of RBE for foreign currency markets. The market as a whole can deviate from economic fundamentals simply because agents do not possess structural knowledge. But this situation is precisely the setting of a currency crisis where small causes trigger an endogenous mechanism that amplifies and propagates panic trading by affecting heterogeneous beliefs. However, such deviations from fundamental equilibrium can only be temporary: rationality in the form of data consistency sooner or later catches up with markets[9]. Currency crises while possibly self-sustaining through secondary effects on beliefs can not persist neither in time nor in structure. However, temporary deviations generating exchange rate fluctuations explain the higher level of asymptotic exchange rate variance under RB.

Agents might also err in trying to forecast the forecasting deviations of others further extending speculative episodes and increasing volatility. Hence, temporary deviations from economic fundamentals can arise from two distinct sources under RBE: erroneous assessments of aggregate forecast H_t and amplification of fundamental shocks. Common to both sources is the propagation mechanism via the feedback effects of realizations, beliefs and trading. Currency crises and excess volatility puzzles are really different sides of the same coin. Both arise from agents' inability to distinguish collective forecasting errors from forecasting deviations, both are propagated by endogenous uncertainty and both lead to excessively high short or long-term variability compared to fundamentals. From a statistical perspective, the parallels go even further: currency crises are the small sample, short term, first moment analog of the asymptotic, second moment effects of endogenous uncertainty.

Periods of wild exchange rate fluctuations - speculative episodes - can not persist under RB. By the data consistency requirement of rationality such episodes have to average out over time

[9] From the proof of Proposition 4.2 rationality restrictions on forecasting errors imply $T^{-1}\sum_{t=0}^{T-1}\zeta_t \to 0$.

which means that they have to be both rare and non-repetitive. Put differently, currency crises can be neither recurrent with respect to their statistical characteristics nor frequent. Otherwise, they might contribute mass to long-term averages under m contradicting rational beliefs. But such episodes are precisely what the non-stationary subjective part P_k stands for in the decomposition of RB: evolutionary, isolated events with big consequences for foreign currency markets.

5. Conclusion

The excess volatility puzzle of exchange rates arises in the context of rational expectations models: for reasonable specifications of exogenous volatility a wide variety of models fail to account for the observed levels of exchange rate variability. Hence, it is tempting to conjecture that assorted exchange rate anomalies and rational expectations are really two sides of the same coin. As a first pass at this hypothesis the present analysis compares the statistical properties of exchange rates under two different expectational paradigms, rational expectations and rational beliefs. The two paradigms differ in their stringency of rationality and informational assumptions. Rational beliefs rely on the weaker rationality condition of data consistency and assume that agents form beliefs based on empirical distributions. Since they nest rational expectations as a special case the excess volatility puzzle can be explored in terms of variance bounds across expectational paradigms.

Rational beliefs easily accommodate diversity of opinion. They introduce just the right amount of rational disagreement into a Lucas two-country cash-in-advance model to endogenously generate uncertainty. Equilibrium exchange rates now convey different signals to agents that will trade even in the absence of new information. As a result, temporary deviations from economic fundamentals become possible. Such foreign exchange crises, however, can not last without violating rationality: exchange rate averages converge to a common asymptotic mean under both expectational paradigms. However, endogenous uncertainty delivers a new volatility generation and amplification mechanism. Consequently, the rational expectations variance provides a lower bound for rational beliefs variances explaining the statistical aspects of the exchange rate volatility problem.

A variance decomposition of exchange rate equilibria reveals the origins of excess volatility. Under rational expectations exchange rate variance is a deterministic function of fundamentals' variance. Structural knowledge precludes any endogenous volatility generation or amplification. If agents know the true structure of the economy they can not rationally disagree about it so that only exogenous volatility drives exchange rate variability. Rational beliefs allow for lack of structural knowledge so that trading reflects uncertainty about the source of volatility, too. With heterogeneous expectations endogenous trading results in rational beliefs variances that are random functions of aggregated forecasts and exogenous variables. Hence, the excess volatility puzzle is a statistical consequence of the severe informational requirements embodied in rational expectations.

In light of the large informational investments by market participants the structural knowledge hypothesis looks quite unreasonable to start with. Trading in response to market movements rather than new information - endogenous uncertainty - is a well established feature of foreign currency markets. In this respect, rational beliefs might offer both a plausible and fruitful alternative to rational expectations. An immediate question is whether foreign currency markets possess structural knowledge and whether this is reflected in the data. Rational beliefs generate testable implications by imposing first and second moment restrictions on exchange rate. Hence, a natural complement to the present theoretical analysis consists in empirically testing the robustness of the link between volatility generation, its transmission and expectational hypothesis.

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Appendix

A. The Rational Beliefs Paradigm

Casting rationality in terms of data consistent beliefs rather than model consistent expectations rephrases the rationality criterion as a learning issue[10]: if agents were given arbitrary but finite amounts of data what subjective beliefs could they reasonably hold? Suppose there exists a finite number K of fundamental variables in the economy so that agents observe $x_t := \left(x_t^1, ..., x_t^K\right) \in X \subseteq \mathbf{R}^K$, where X is the state space. The economic system is represented by a probability space $(\Omega, \mathcal{F}, \Pi, T)$ on the non-negative integers with its natural filtration[11] $(\Omega = X^\infty)$ and T is the Bernoulli (shift) operator. For a sample point $(x)^t := (x_t, x_{t+1}, ...)$ starting at t the operator $T : \Omega \to \Omega$ shifts realizations forward by one period so that $(x)^{t+1} = T(x)^t$ and

$$(x)^t = T^t x \text{ for } t \geq 0.$$

Let $T^{-n}S$ be the preimage of S under T, i.e., $T^{-n}S := \{x : T^n x \in S\}$ for $S \in \mathcal{F}$.

By assumption, agents do not know the true structure of the economy (Π) but analyze their environment by computing frequencies on finite data sets. Trying to discover the true probability of an event, $\Pi\{S\}$, $S \in \mathcal{F}$, agents calculate

$$m_n(S)(x) := n^{-1} \sum_{k=0}^{n-1} 1_S\left(T^k x\right)$$

which is the relative frequency of the dynamical system visiting S given that it started at x. However, they can only gain knowledge about Π to the extent that the dynamic system exhibits repetitive regularities. Hence, a prerequisite for learning, meaningful inference and rationality is that $n \to \infty \lim m_n(S)(x)$ exists which leads to the concept of stability:

Definition A.1 (Stability). *An economic system is said to be* stable *if probabilities of finite dimensional events computed as frequencies converge; i.e., $(\Omega, \mathcal{F}, \Pi, T)$ is stable if for all cylinders $S \in \mathcal{C} \subset \mathcal{F}$, there exists* Π-a.s.

$$n \to \infty \lim m_n(S)(x) := \tilde{m}(S)(x).$$

By the convergence of empirical distributions to $\tilde{m}$ on $\mathcal{C}$, the latter can be extended to a measure m on $\mathcal{F}$, the empirical measure, under which the economic system is stationary:

Proposition A.2 (Empirical Measure). *If $(\Omega, \mathcal{F}, \Pi, T)$ is stable, the set function $\tilde{m}$ on the collection of cylinders $\mathcal{C} \subset \mathcal{F}$ has a unique extension to a probability measure $m(\cdot)(x)$ on $(\Omega, \mathcal{F})$ such that $(\Omega, \mathcal{F}, m, T)$ is stationary.*

Proof. Using the topological properties of $\mathbf{R}^\infty$ [5, Billingsley 1995]: 29 proves that every finitely additive probability measure such as $\tilde{m}$ on the cylinder field $\mathcal{C}(\mathbf{R}^\infty)$ is countably additive so that Caratheodory's Extension Theorem applies. The stationarity of $(\Omega, \mathcal{F}, m, T)$ is an immediate consequence of stability: for $S \in \mathcal{C}$, $\tilde{m}\left(T^{-1}S\right)(x) = \tilde{m}(S)(x)$ and, hence, $m\left(T^{-1}F\right)(x) = m(F)(x)$ for $F \in \mathcal{F}$. ■

[10] The material is loosely based on [19, Kurz 1991] and [21, Kurz 1996].

[11] The *natural filtration* is the filtration generated by the respective random sequence itself, i.e., $\mathcal{F}_t := \sigma\{x_s : s \leq t\} \uparrow \mathcal{F} = \sigma\left\{\bigcup_t \mathcal{F}_t\right\}$.

Economic theory usually equates forming a belief with calculating conditional probabilities under appropriate rationality restrictions in order to avoid logical inconsistency. In the absence of structural knowledge rational beliefs cast rationality in terms of three data consistency requirements:

Definition A.3 (Rational Belief). *A probability measure Q on the underlying economic system $(\Omega, \mathcal{F}, T)$ is a* rational belief *if it satisfies*

1. *stability: the economic system $(\Omega, \mathcal{F}, Q, T)$ is stable;*
2. *data compatibility: Q generates the empirical measure, i.e. for all cylinders $S \in \mathcal{C} \subset \mathcal{F}$,*

$$m_Q(S) := n \to \infty \lim n^{-1} \sum_{k=0}^{n-1} Q\left(T^{-k}S\right) = m(S)$$

   so that $m_Q = m$ on finite dimensional sets;

3. *non-degeneracy: for $S \in \mathcal{F}$, $m(S) > 0 \Rightarrow Q(S) > 0$.*

The first condition insures meaningful inference, a prerequisite for statistical agreement of belief and observation. The second one requires a belief Q to agree with the observed realization in the sense of empirical distribution learning and, ultimately, hypothesis testing. Finally, one has to impose a continuity requirement: an event with positive empirical probability must happen infinitely often. So, from the perspective of an arbitrary date, its occurrence should not be excluded requiring a subjective assessment with positive probability.

Empirical observation and rational beliefs are related by the following key result of [19, Kurz 1991]:

Theorem A.4 (Characterization of RB). *A rational belief Q can be represented as*

$$Q = \lambda m + (1 - \lambda) P \tag{A.1}$$

where m is the stationary empirical probability, P is a probability measure and $0 < \lambda \leq 1$. Moreover, m and P are mutually singular, the economic system is stable under both Q and P, and $m_Q = m = m_P$.

Proof. For details refer to [19, Kurz 1991]. ■

If m represents the stationary, learnable characteristics of the economic system, P corresponds to the evolutionary, non-stationary element of the agent's model. By $m_Q = m = m_P$ timing matters: non-stationary, evolutionary episodes have to be rare since they need to vanish by stability requirements. P allows slight deviations from the common knowledge m at different, possibly infinite points of time that only have to average out over time. Similarly informed agents can now rationally disagree about the nature of fluctuations in a dynamic economy.

B. Proofs

B.1. Proposition 3.2

Proof. By stability, there exists $\alpha := T \to \infty \lim T^{-1} \sum_{t=0}^{T-1} \alpha_t$, $x := T \to \infty \lim T^{-1} \sum_{t=0}^{T-1} x_t$ and, without loss of generality, $T \to \infty \lim T^{-1} \sum_{t=0}^{T-1} \xi_t = 0$ from the stability conditions on η_t and ϵ_t.

For notational simplicity let x_t and ξ_t be independent. Proposition 2.1 shows that agents form rational beliefs forecasts

$$E_{Q_k}\left[e_t|\,\mathcal{G}_t\right] = \lambda_k E_m\left[e_t|\,\mathcal{G}_t\right] + (1-\lambda_k)\, E_{P_k}\left[e_t|\,\mathcal{G}_t\right].$$

Letting $e_t^m(\mathcal{G}_t) := E_m\left[e_t|\,\mathcal{G}_t\right]$ and $e_t^{P_k}(\mathcal{G}_t) := E_{P_k}\left[e_t|\,\mathcal{G}_t\right]$, Equation (3.1) yields the fundamental exchange rate equation

$$e_t = \left(a_t + x_t^{\mathrm{T}} h\right) + b\int_0^1 \left[\lambda_k e^m(\mathcal{G}_t) + (1-\lambda_k)\, e_t^{P_k}(\mathcal{G}_t)\right] dk + \xi_t. \tag{B.1}$$

The result now follows from calculating $e_t^m(\mathcal{G}_t)$ and $e_t^{P_k}(\mathcal{G}_t)$ under Q_k in Equation (3.1) exploiting data consistency and substituting the resulting expressions back into the preceding expression.

By definition the conditional expectation $E_\mu\left[e_t|\,\mathcal{G}_t\right]$ is a $\mathcal{G}_t$-measurable random variable satisfying $\int_G E_\mu\left[e_t|\,\mathcal{G}_t\right] d\mu = \int_G e_t d\mu$, $\forall G \in \mathcal{G}_t \subset \mathcal{F}$. But data consistency of RB implies that

$$m_{Q_k}(S) := n \to \infty \lim n^{-1} \sum_{k=0}^{n-1} Q_k\left(T^{-k} S\right) = m(S)$$

by Definition A.3 so that by standard arguments[12] and properties of conditional expectations

$$n^{-1} \sum_{s=t}^{t+n-1} E_{Q_k}\left[e_s|\,\mathcal{G}_s\right] Q_k\left(T^{-s} S\right) \to E_m\left[e_t|\,\mathcal{G}_t\right]$$

and, similarly, $n^{-1} \sum_{s=t}^{t+n-1} e_s Q_k\left(T^{-s} S\right) \to E_m\left[e_t|\,\mathcal{G}_t\right]$. Observability of x_t and stability give the common, stationary forecast as

$$e_t^m(\mathcal{G}_t) = (1-b)^{-1}\left(a + x_t^{\mathrm{T}} h\right).$$

Defining the subjective, non-stationary forecast as

$$e_t^P(\mathcal{G}_t) := \int_0^1 (1-\lambda_k)\, e_t^{P_k}(\mathcal{G}_t)\, dk$$

and $\lambda := \int_0^1 \lambda_k dk$ the rational beliefs equilibrium price process becomes

$$e_t(\mathcal{G}_t) = \left(a_t + x_t^{\mathrm{T}} h\right) + b\lambda (1-b)^{-1}\left(a + x_t^{\mathrm{T}} h\right) + b e_t^P(\mathcal{G}_t) + \xi_t. \qquad \blacksquare$$

B.2. Proposition 3.3

Proof. From the respective equilibrium definitions in Equations (3.2) and (3.3) one has

$$\begin{aligned} e_t^{REE} &= (1-b)^{-1}\left(a_t + x_t^{\mathrm{T}} h\right) + \xi_t \\ e_t^{RBE} &= \left(a_t + x_t^{\mathrm{T}} h\right) + b\lambda (1-b)^{-1}\left(a + x_t^{\mathrm{T}} h\right) + b\int_0^1 (1-\lambda_k)\, p_t^{P_k}(\mathcal{G}_t)\, dk + \xi_t; \end{aligned}$$

[12] Prove the result first for $e_t = 1_F$, then for a simple random variable e_t, approximate positive and negative parts of e_t by simple, monotonically converging sequences and use the Monotone Convergence Theorem.

but stability implies $T^{-1}\sum_{t=0}^{T-1} a_t \to a$, $T^{-1}\sum_{t=0}^{T-1} x_t \to x$ and $T \to \infty\lim T^{-1}\sum_{t=0}^{T-1} \xi_t \to 0$ so that

$$T \to \infty\lim T^{-1}\sum_{t=0}^{T-1} e_t^{REE} = T \to \infty\lim T^{-1}\sum_{t=0}^{T-1}\left\{(1-b)^{-1}\left(a_t + x_t^{\mathrm{T}}h\right) + \xi_t\right\} = (1-b)^{-1}\left(a + x^{\mathrm{T}}h\right).$$

Similarly, using again the stability conditions and ergodic properties of m

$$\begin{aligned}
T \to \infty\lim T^{-1}\sum_{t=0}^{T-1} e_t^{RBE} &= T \to \infty\lim T^{-1}\sum_{t=0}^{T-1}\left\{\left(a_t + x_t^{\mathrm{T}}h\right) + b\lambda(1-b)^{-1}\left(a + x_t^{\mathrm{T}}h\right)\right\} \\
&\quad + T \to \infty\lim T^{-1}\sum_{t=0}^{T-1} b\int_0^1 (1-\lambda_k)\, e_t^{P_k}(\mathcal{G}_t)\, dk \\
&= (1-b)^{-1}\left(a + x^{\mathrm{T}}h\right)(1 - b(1-\lambda)) \\
&\quad + T \to \infty\lim T^{-1}\sum_{t=0}^{T-1} b\int_0^1 (1-\lambda_k)\, e_t^{P_k}(\mathcal{G}_t)\, dk \\
&= (1-b)^{-1}\left(a + x^{\mathrm{T}}h\right)
\end{aligned}$$

because

$$\begin{aligned}
T \to \infty\lim T^{-1}\sum_{t=0}^{T-1}\int_0^1 (1-\lambda_k)\, e_t^{P_k}(\mathcal{G}_t)\, dk &= \int_0^1 (1-\lambda_k)\left[T \to \infty\lim T^{-1}\sum_{t=0}^{T-1} e_t^{P_k}(\mathcal{G}_t)\right] dk \\
&= (1-b)^{-1}\left(a + x^{\mathrm{T}}h\right)\int_0^1 (1-\lambda_k)\, dk \\
&= (1-b)^{-1}\left(a + x^{\mathrm{T}}h\right)(1-\lambda)
\end{aligned}$$

where the first equality is a consequence of the Monotone Convergence Theorem[13] and the second one follows from Proposition 2.1: since $m = m_\Pi = m_{Q_k}$, $m_{P_k} = m$. Now use the stability conditions and the definition of λ. ■

B.3. Proposition 4.1

Proof. Under RE, $Q_k = \Pi$, $\forall k$ so that by the ergodic properties of m under RB[14] $e = T \to \infty\lim T^{-1}\sum_{t=0}^{T-1} e_t^{REE} = E_m\left[e_t^{REE}\right] = E\left[e_t^{REE}\right]$ and hence

$$T^{-1}\sum_{t=0}^{T-1}\left(e_t^{REE} - e\right)^2 \to E_m\left[e_t^{REE} - e\right]^2 = E\left[e_t^{REE} - e\right]^2 := \sigma_{REE}^2$$

by the same ergodic argument. Since

$$\left(e_t^{REE} - e\right) = (1-b)^{-1}\left\{(a_t - a) + (x_t - x)^{\mathrm{T}}h\right\} + \xi_t$$

[13] [5, Billingsley 1995]: 208.

[14] See Proposition 3.3. Since stability is defined in terms of finite dimensional empirical distributions, it requires that the limits of all sample moments converge. But $(\Omega, \mathcal{F}, m(\cdot)(x), T)$ can be shown to be ergodic so that the limits are the second moments of the random variables under the empirical measure.

the convergence of sample moments under stability and the uncorrelatedness assumptions on the random variables imply that

$$T \to \infty \lim T^{-1} \sum_{t=0}^{T-1} \left(e_t^{REE} - e\right)^2 = E\left\{(1-b)^{-2}\left[(a_t - a)^2 + \left((x_t - x)^{\mathrm{T}} h\right)^2\right] + \xi_t^2\right\}.$$

whence

$$\sigma_{REE}^2 = (1-b)^{-2}\left(\sigma_a^2 + \sigma_x^2\right) + \sigma_\xi^2. \quad \blacksquare$$

B.4. Proposition 4.2

Proof. For agent k forecasts and realization are related by $e_t = e_t^{Q_k}(\mathcal{G}_t) + \nu_t^k$ with forecast errors ν_t^k that, by stability, can be taken to satisfy $T^{-1}\sum_{t=0}^{T-1} \nu_t^k \to 0$. This, in turn, implies that, under m,

$$T^{-1} \sum_{t=0}^{T-1} e_t^{Q_k}(\mathcal{G}_t) \to e = (1-b)^{-1}\left(a + x^{\mathrm{T}} h\right) \tag{B.2}$$

by Proposition 3.3 and, hence, forecasts $e_t^{Q_k}(\mathcal{G}_t)$ satisfy assorted orthogonality restrictions. From Equation (3.1)

$$e_t - e = (1-b)^{-1}\left\{(a_t - a) + (x_t - x)^{\mathrm{T}} h\right\} + b\left\{\int_0^1 e_t^{Q_k}(\mathcal{G}_t)\, dk - (1-b)^{-1}\left(a_t + x_t^{\mathrm{T}} h\right)\right\} + \xi_t$$

whence it follows that for aggregate forecasting functions $H_t := \int_0^1 e_t^{Q_k}(\mathcal{G}_t)\, dk$

$$T^{-1} \sum_{t=0}^{T-1} (e_t - e)^2 \to \sigma^2 = (1-b)^{-2}\left[\sigma_a^2 + \sigma_x^2 + b^2\left(\sigma_H^2 + \sigma_a^2 + \sigma_x^2\right)\right] + \sigma_\xi^2.$$

Now, Proposition 4.1 yields $\sigma^2 = \sigma_{REE}^2 + (1-b)^{-2} b^2\left(\sigma_H^2 + \sigma_a^2 + \sigma_x^2\right)$.

In the special rational beliefs equilibrium of Equation (3.4) where $Q_k = m$, $\forall k$ implies $\lambda_k = 1 = \lambda$

$$\left(e_t^{RBE^*} - e\right) = (a_t - a) + (1-b)^{-1}(x_t - x)^{\mathrm{T}} h + \xi_t$$

so that by the same arguments as above

$$\sigma_{RBE^*}^2 = \sigma_a^2 + (1-b)^{-2}\sigma_x^2 + \sigma_\xi^2;$$

but $b = -\frac{\beta}{\gamma} < 0$ so that $\sigma_{REE}^2 < \sigma_{RBE^*}^2$ by $(1-b)^{-2} < 1$. $\blacksquare$

B.5. Proposition 4.3

Proof. This is a direct consequence of Propositions 4.1 and 4.2:

$$\sigma_{REE}^2 = (1-b)^{-2}\left(\sigma_a^2 + \sigma_x^2\right) + \sigma_\xi^2 \leq \sigma_{REE}^2 + (1-b)^{-2} b^2\left(\sigma_H^2 + \sigma_a^2 + \sigma_x^2\right) = \sigma_{RBE}^2$$

by $(1-b)^{-2} b^2\left(\sigma_H^2 + \sigma_a^2 + \sigma_x^2\right) \geq 0$. The second inequality $\sigma_{RBE}^2 \geq \sigma^2(\mathcal{G}_t)$ follows from the fact that stability places no restrictions on (small) sample moments. $\blacksquare$

B.6. Corollary 4.4

Proof. By the usual stability arguments $T^{-1}\sum_{t=0}^{T-1}\sum_{l=0}^{n}\theta_{t-l}^{k}e_{t-l-1}\to a$ so that

$$e_t=(1-b)^{-1}\left(a_t+x_t^{\mathrm{T}}h\right)+b\,(1-b)^{-1}\left[\sum_{l=0}^{n}\left(\int_0^1\theta_{t-l}^{k}dk\right)e_{t-l-1}-a_t\right]+\xi_t$$

implies for $\theta_t:=\sum_{l=0}^{n}\left(\int_0^1\theta_{t-l}^{k}dk\right)e_{t-l-1}dk$

$$T^{-1}\sum_{t=0}^{T-1}(e_t-e)^2\to\sigma^2=(1-b)^{-2}\left(\sigma_a^2+\sigma_x^2\right)+b^2\,(1-b)^{-2}\left(\sigma_\theta^2+\left(1-\frac{2}{b}\right)\sigma_a^2\right)+\sigma_\xi^2.\ \blacksquare$$

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ISSN 1436 - 6053

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